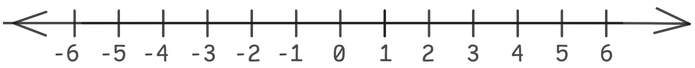
Math 3150 **Exam 1A Solutions**, Feb 2026, 12 problems, 75 minutes

Problems 1-2 on this page refer to the following sets:

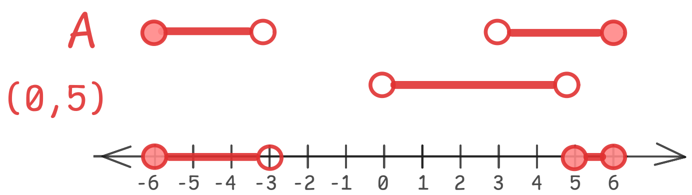
$$A = \{x \in \mathbb{R} : 9 < x^2 \leq 36\}, \qquad B = \{x \in \mathbb{N} : x \equiv 0 \pmod 3\}, \qquad C = \{19n : n \in \mathbb{N}\}$$

Problem 1A. Graph the set $A - (0,5)$ on the number line:



Answer

Observe that taking the square roots of $9 < x^2 \leq 36$ results in two different intervals: $3 < x \leq 6$ and $-3 > x \geq -6$. Therefore the difference $A - (0,5)$ only removes the positive part of these intervals, leaving the answer to be $[-6,-3) \cup [5,6]$:



Problem 1B. Find the cardinality of $B \cap [0,100]$.

Answer

Since B consists of multiples of 3, we must find how many multiples of 3 are between 0 and 100:

$$0 \leq 3m \leq 300 \Rightarrow 0 \leq m \leq 100/3 = 33.333\ldots \Rightarrow m = 0, 1, 2\ldots, 33.$$

So there are precisely 34 values of m which correspond to the multiples $3m$ that lie between 0 and 100. Answer: $|B \cap [0,100]| = 34$.

Problem 2A. Find the cardinality of $(B \cup C) \cap [0,100]$.

Answer

This question asks for how many natural numbers between 0 and 100 are multiples of 3 or multiples of 19.

From **Problem 1B**, we found that there are 34 multiples of 3 in $[0,100]$.

There are 6 multiples of 19 in $[0,100]$, namely, $0 \times 19, 1 \times 19, 2 \times 19, 3 \times 19, 4 \times 19, 5 \times 19$.

The only overlap between these two counts are 0 and 3×19.

So by the principle of inclusion-exclusion (PIE),

$$|(B \cup C) \cap [0,100]| = 34 + 6 - 2 = 38.$$

Problem 2B. List the elements of $B \cap [1004, 1010]$.

Answer

Here we must list which numbers $1004, 1005, 1006, \ldots, 1010$ are multiples of 3. We can either use long division to check, or use the divisibility rule for 3 to see that

$$1004 \equiv 1 + 0 + 0 + 4 \equiv 5 \equiv 2 \pmod 3$$

so $1005 \equiv 2 + 1 \equiv 0 \pmod 3$ and $1005 + 3 = 1008 \equiv 0 \pmod 3$. These are the only multiples of 3 in the range $[1004, 1010]$.

Answer: $B \cap [1004, 1010] = \{1005, 1008\}$.

Problem 3A. Convert 101_5 to base 10.

Answer

$101_5 = 1 \times 5^2 + 0 \times 5 + 1 = 25 + 1 = 26$.

Problem 3B. Convert 101_5 to base 3.

Answer

From Problem 3A, we saw that $101_5 = 26$. Next we use repeated division by 3 to find what 26 is in base 3:

$$\begin{aligned} 26 &= 8 \times 3 + 2 \\ 8 &= 2 \times 3 + 2 \\ 2 &= 0 \times 3 + 2 \end{aligned}$$

Therefore $101_5 = 26 = \boxed{222_3}$.

Problem 4. Convert the RGB color code $(169, 170, 172)$ to a HEX color code.

Answer

Observe that $169 = 10 \times 16 + 9 = A9_{16}$ so $170 = AA_{16}$ and $172 = AA_16 + 2 = AC_{16}$. Answer: $\#A9AAAC$.

Problem 5. Compute $\gcd(603733, 2743)$, given that the Euclidean algorithm starts with the following first line of computation:

$$603733 = 220 \times 2743 + 273$$

Answer

We continue the above Euclidean algorithm to get:

$$\begin{aligned} 603733 &= 220 \times 2743 + 273 \\ 2743 &= 10 \times 273 + 13 \\ 273 &= 21 \times 13 + 0 \end{aligned}$$

so $\gcd(603733, 2743) = 13$.

Problem 6. Find integers $m, n \in \mathbb{Z}$ that satisfy the equality

$$\gcd(603733, 2743) = 603733m + 2743n$$

Answer

Answer: We use the Euclidean algorithm above to get

$$\begin{array}{rr} -10 \times & [\underline{603733} = 220 \times \underline{2743} + \underline{273}\ ] \\ + & \underline{2743} = 10 \times 273 + 13 \\ \hline -10 \times \underline{603733} + 1 \times \underline{2743} & = -2200 \times \underline{2743} + \underline{13} \end{array}$$

$$\Rightarrow \qquad -10 \times \underline{603733} + 2201 \times \underline{2743} = \underline{13}$$

So $m = -10$ and $n = 2201$ is a valid solution.

Problem 7A. Compute the (positive) remainder of $39 \times 38 \times 37$ divided by 12.

Answer

Observe that $39 \equiv 39 - 36 \equiv 3$ so $38 \equiv 2$ and $37 \equiv 1 \pmod{12}$. Therefore

$$39 \times 38 \times 37 \equiv 3 \times 2 \times 1 \equiv 6 \pmod{12}$$

and 6 is the positive remainder that results when $39 \times 38 \times 37$ is divided by 12.

Problem 7B. Compute the (positive) remainder of 237^4 divided by 23.

Answer

Observe that $237 \equiv 7 \pmod{23}$ so:

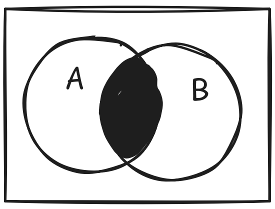
$$237^4 \equiv 7^4 \equiv (7^2)^2 \equiv (49)^2 \equiv (3)^2 \equiv 9 \pmod{23}$$

so 9 is the positive remainder that results when 237^4 is divided by 23.

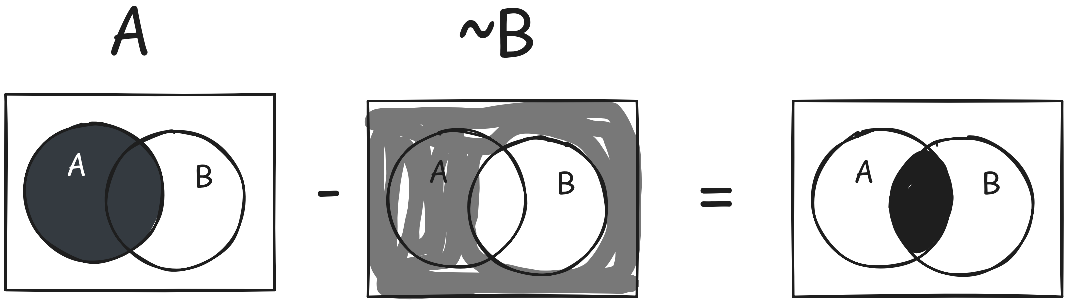
Problem 8A. Draw the Venn diagram for $A - (\sim B)$.

Answer

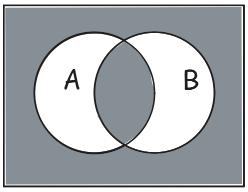
One way to see this is that $A - (\sim B) = A \cap \sim (\sim B) = A \cap B$ so the answer is:



Another way to get the answer is to draw both A and $\sim B$ out and do the set difference:



Problem 8B. Put the following Venn diagram into set notation.



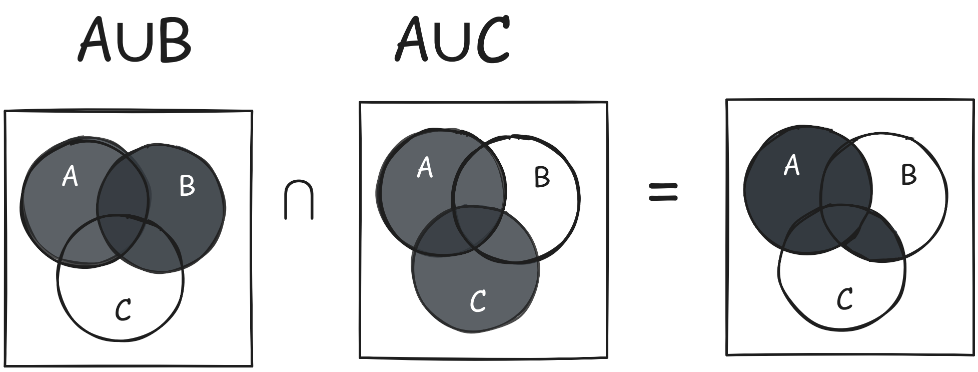
Answer

The outer shaded region is the complement of A union B, or $\sim (A \cup B)$. The inner shaded region is the intersection of A and B, or $A \cap B$. We want both shading, so we take their union:

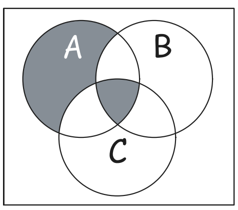
$$(\sim (A \cup B)) \cup (A \cap B)$$

Problem 9A. Draw the Venn diagram for $(A \cup B) \cap (A \cup C)$.

Answer



Problem 9B. Put the following Venn diagram into set notation.



Answer

The middle part is the triple intersection $A \cap B \cap C$. The upper left shaded region is $(A - B) - C$. Since we want to keep both shadings, the set can be written as

$$((A - B) - C) \cup (A \cap B \cap C)$$

Problem 10. Find a base b such that $12_b + 10_{10} = 100_b$.

Answer

We can expand this out as $(1 \times b + 2) + 10 = (1 \times b^2 + 0 \times b + 0)$ or $b + 2 + 10 = b^2$.

This rearranges to $0 = b^2 - b - 12 = (b-4)(b+3)$. The only positive solution is $b = 4$.

Problem 11. Compute the remainder of the division $121111121111121111 \div 1001$.

Answer

Similar to the patterns for divisibility by 11 (in-class) and by 101 (practice problems):

$$\begin{aligned} 121111121111121111 &\equiv 121 \times 1000^5 + 111 \times 1000^4 + 121 \times 1000^3 + 111 \times 1000^2 + 121 \times 1000 + 111 \\ &\equiv 121 \times (-1)^5 + 111 \times (-1)^4 + 121 \times (-1)^3 + 111 \times 1000^2 + 121 \times (-1) + 111 \\ &\equiv -121 + 111 - 121 + 111 - 121 + 111 \\ &\equiv (-121 + 111) + (-121 + 111) + (-121 + 111) \\ &\equiv (-10) + (-10) + (-10) \\ &\equiv -30 \\ &\equiv 1001 - 30 = 971 \pmod{1001} \end{aligned}$$

Problem 12. Convert the number 2026_8 from base 8 to base 16.

Answer

Since 8 and 16 are powers of 2, it is easier to use base 2 as an intermediate base to convert instead of through base 10.

$2026_8 = 010|000|010|110_2 = 0100|0001|0110_2 = 416_2$
